APPENDIX

Several results from application of the two yield criteria used above with two common stress states are of general interest and will be developed below. In the following, normal stresses are positive if tensile, pressure P is always a positive quantity, shear stress τ are always positive, and T_{ij} is a deviatoric stress in the j direction caused by a force in the i direction. To discriminate between the criteria but display the results for quick comparison, the equations pertaining to one or the other criteria will be placed on a particular side of a centre line, following the example

Modified von Mises criterion	Modified Mohr–Coulomb criterion	
$\tau_{oct}=\tau_s-\mu T_m$	$\tau_{max}=\tau_0-\mu'\sigma_m$	(5)

(I) For a uniaxial test with $\sigma_1=T_{11}P$ and $\sigma_2=\sigma_3=-P$, the stress state requires that $\sigma_m=(T_{11}/3)-P$ . . . . (6)

By definition,

$$\tau_{oct}=\pm(\sqrt{\tfrac{2}{3}})T_{11} \qquad \tau_{max}=\pm(\tfrac{1}{2})T_{11} \qquad (7)$$

where the upper sign refers to tensile tests and the lower to compression tests. Combining eqn. (6) with eqn. (7),

$$\tau_{oct}=\pm\sqrt{2}(\sigma_m+P) \qquad \tau_{max}=\pm(\tfrac{3}{2})(\sigma_m+P)$$

and then imposing the yield criteria, eqns. (3),

$$T_{11}{}^{Y}=(3/\mu\pm\sqrt{2})(\tau_s+\mu P) \qquad T_{11}{}^{Y}=[3/\mu'\pm(\tfrac{3}{2})](\tau_0+\mu' P)$$

The ratios of compressive-to-tensile true shear-yield stresses would then be

$$\frac{T_{11}{}^{Y}(C)}{T_{11}{}^{Y}(T)}=\frac{\mu+\sqrt{2}}{\mu-\sqrt{2}} \qquad \frac{T_{11}{}^{Y}(C)}{T_{11}{}^{Y}(T)}=\frac{\mu'+(\frac{3}{2})}{\mu'-(\frac{3}{2})}. \qquad (8)$$

These can be rearranged to solve for the parameter describing the pressure dependence,

$$\mu=\sqrt{2}\,\frac{T_{11}{}^{Y}(C)+T_{11}{}^{Y}(T)}{T_{11}{}^{Y}(C)-T_{11}{}^{Y}(T)} \qquad \mu'=(\tfrac{3}{2})\frac{T_{11}{}^{Y}(C)+T_{11}{}^{Y}(T)}{T_{11}{}^{Y}(C)-T_{11}{}^{Y}(T)}$$

which for those uniaxial tests sets,

$$\mu=(2\sqrt{2})\mu'/3.$$

Equating the expressions for $T_{11}{}^{Y}$ of the two yield criteria and substituting from the last equation results in

$$\tau_s=(2-\sqrt{2})\tau_0/3$$

(II) For pure shear biaxial tests,

$$\sigma_{\mathrm{m}} = -P$$

and, by definition,

$$\tau_{\mathrm{oct}}(-\sqrt{\tfrac{2}{3}})T_{12}(S) \qquad \tau_{\max} = T_{12}(S)$$

Imposing the respective yield criteria, eqns. (3),

$$T_{12}{}^{\mathrm{Y}}(S) = (-\sqrt{\tfrac{3}{2}})(\tau_{\mathrm{s}} + \mu P) \qquad T_{12}{}^{\mathrm{Y}}(S) = (\tau_0 + \mu' P). \quad . \quad . \quad . \quad (9)$$

At atmospheric pressure, where $\sigma_{\mathrm{m}} = 0$, which is the common situation for shear tests,

$$\tau_{\mathrm{s}} = \tau_0 - \sqrt{(\tfrac{2}{3})}$$

Equating the expressions for $T_{12}{}^{\mathrm{Y}}(S)$ of the two yield criteria and substituting from the last equation results in

$$\mu = \mu' \sqrt{(\tfrac{2}{3})}$$

The equations all revert to accepted forms at atmospheric pressure ($\sigma_{\mathrm{m}} = 0$) or for pressure-independent yield stresses ($\mu = 0$). Note that although μ and τ_{s} refer to any stress state, μ' and τ_0 vary with the stress state.

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